Tuesday, February 03, 2015 12:55 PM

Deep Thoughts

From considerations of Newtonian gravity and Newtonian mechanics we observe the Weak Equivalence Principle (WEP): $m_G = m_I$

$$\vec{F}_G = m_G \underbrace{\vec{g}}_{\underbrace{\sum_i \frac{G m_{G i} (\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}}_{\text{Describes only one force, gravity.}}} \qquad \underbrace{\sum \vec{F}}_{\text{Incorporates all forces on object!}} = m_I \vec{a}$$

Describes only one force, gravity. → If only gravity is present:

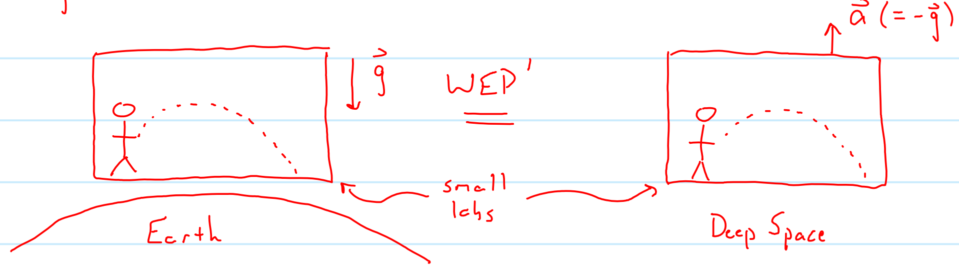
$\underbrace{\vec{a} = \vec{g}}$ (ind. of m)

(contrast w/ E&M $\vec{a} = \frac{q}{m}\vec{E}$)

Einstein took the WEP and extended it to arrive at GR. Before extending it, it helps to rewrite it.

WEP': $\vec{a} = \vec{g}$ For massive objects and external gravity, a uniform external gravitational field is indistinguishable from a uniform acceleration.

Consider projectile motion:



Tuesday, February 03, 2015 1:05 PM

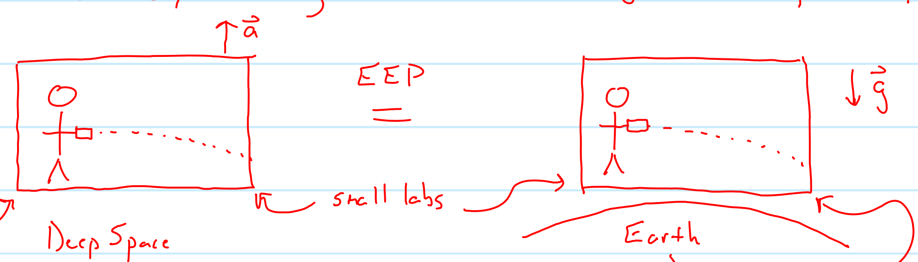
Deeper Thoughts

Now extend the WEP' to the

Einstein Equivalence Principle (EEP): For *any* object and *any* force (except internal gravitation) a uniform external gravitational field is indistinguishable from uniform acceleration.

What does this get us? For starters it says something nontrivial about light ($m=0$, $E \neq m$).

Consider a flashlight:



Nothing weird here (an inertial observer would see ---)

This is weird since $m=0$ for light!

Tuesday, February 03, 2015 1:12 PM

Deeperer Thoughts

The EEP is sufficient to motivate the basics of GR. However we can go further.

Strong Equivalence Principle (SEP): For any object and any force a uniform external gravitational field is indistinguishable from uniform acceleration.

What does this get us? It singles out GR over alternative theories (Brans-Dicke, etc.)

Tuesday, February 03, 2015 1:15 PM

Deeperest Thoughts

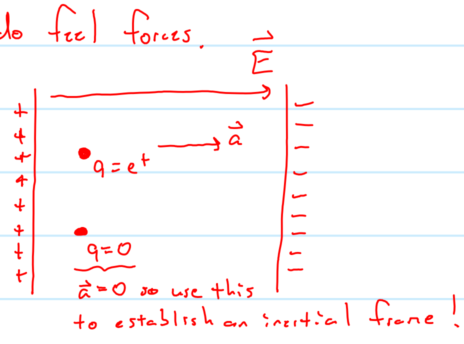
In both Newtonian mechanics and SR, the concept of inertial frames played crucial roles. In NM, inertial frames are those in which we can use N2L. In SR, we formulate the framework based on the tenet that physics (including c) is unchanged when viewed from different inertial frames.

But how do we define/find inertial frames?

Newton's First Law: In an inertial frame an object experiencing no net force will move w/ a constant velocity.

So... aside from trying to balance multiple forces, the easiest way to establish an inertial frame is to find an object which is unaffected by the forces present. Then this object defines an inertial frame in which we can use N2L to describe the motion of objects that do feel forces.

This works well for E+M:

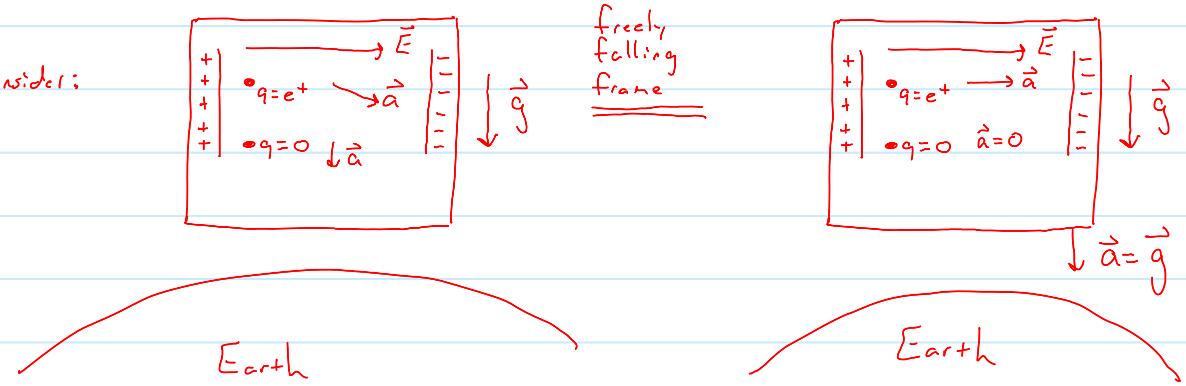


The problem with gravity is that the EEP tells us that there are no objects which are unaffected by gravity, hence there are no naturally defined inertial frames.

One of Einstein's most significant insights is that we can re-establish the Newtonian construction of inertial frames if we allow ourselves to disregard gravity as a "force" in the way we think of all other interactions. Once you are willing to single out gravity, the trick is to "get rid of it". How? Jump off a cliff!

Einstein realized that a reference frame which is "freely falling" under the influence of external gravity is our best scenario for establishing inertial frames.

Consider:



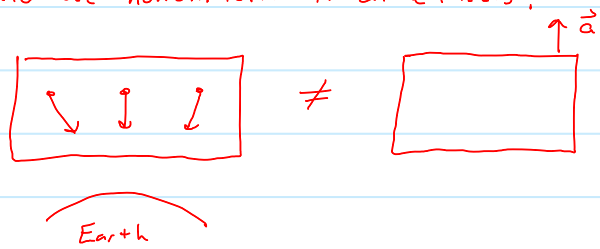
Moreover, in the presence of gravity, by going to a freely-falling frame we actually mimic the behavior we would expect to see in deep-space (w/out gravity). This is how we "get rid of gravity" by jumping off a cliff.

Note: The key to this working is the universality of how gravity influences objects. Everything experiences $\vec{a}=\vec{g}$ regardless of mass! This type of universal influence is one more reason to suspect that gravity is less like the other forces and instead tied to a universal feature like... spacetime!

Tuesday, February 03, 2015 5:00 PM

One Big Fat Caveat!

In all of our equivalence principles we have been careful to use a uniform gravitational field (since the cancelling accelerations are uniform). If our lab was too big we would see nonuniform tidal effects:



So we need to restrict ourselves to small regions of space and time.
This leads to our final refinement of the EEP:

> Experiments performed in a small, freely-falling lab over a short time give results that are indistinguishable to those in an inertial frame in empty space.

This might sound like a limitation, but it actually leads us to the definition of manifolds (curved spaces that are locally flat) as the spacetimes upon which GR can exist.

SR: The laws of physics are invariant under transformations connecting inertial frames, and spacetime is isotropic in space and homogenous in space-time. $\Rightarrow P^4 = ISO(1,3)^{\uparrow} \ltimes T^4$

GR: The laws of physics are invariant under diffeomorphisms of spacetime } tells us how things behave on curved space, i.e. respond to gravity

and the connection (gauge-field) facilitating this invariance should be rendered dynamical by the introduction of an invariant field strength term. tells us how curvature arises dynamically from sources

It will take time to digest all of this, especially the second half, but the first half will seem similar to what we did in SR, i.e. identify transformations and then demand laws be expressed in terms of true tensors under those transformations.

To get an idea of why the transformations in GR will be more complicated, consider SR where we can coordinatize $\mathbb{M}^4$ globally w/ (t,x,y,z) and metric $\eta_{\mu\nu}$. This means that the Lorentz transformations (actually Poincaré) act the same everywhere, and it is in these coordinates that physical laws take their simplest form (preferred coordinates).

However we learned from the EEP that when gravity is present, we will at best be able to describe things in terms of flat space physics over a small region. We will not be able to extend this to a global symmetry of the entire space. Moreover, since each region can be made flat exclusively of the others, there is no preferred coordinate system, So we are going to need a much more general set of transformations that exhibit no prefered coordinates. $\Rightarrow$ "General coordinate transformations" or "General Covariance"

But coordinates are really not physical, so much of this we should be able to consider in a coordinate independent form.

Friday, February 06, 2015 8:20 AM

Einstein Equivalence Principle: Experiments performed over a short time in a small freely-falling lab give results indistinguishable from those obtained in an inertial frame in empty space.

This along with the absence of any gravitationally "neutral" objects lead us to define "inertial" frames as freely-falling frames (FFF) since:

$$\underbrace{\boxed{\text{Lab}}\downarrow\vec{g} = \boxed{\text{Lab}}\uparrow\vec{a}}_{\text{uniform}} \Rightarrow \boxed{\text{Lab}}\downarrow\vec{g} \underbrace{- \boxed{\text{Lab}}\uparrow\vec{a}}_{+\,\boxed{\text{Lab}}\downarrow\vec{a}\ \text{freely-falling}} = \boxed{\text{Lab}}$$

Small labs and short times are required to ensure approximate uniformity of $\vec{g}$, even if it changes w/ position.

2 important implications:

1. In an FFF we can use SR since this correctly describes physics in deep space (no gravity).
2. When looking for what type of spacetimes are allowed in GR, they must be locally flat, i.e. a manifold.

Nomenclature:

$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$ Euclidean Space (in $x, y, z, \ldots$)

$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$ Minkowski Space (in $t, x, y, \ldots$)

} Special Cases

Riemannian $\quad g_{\mu\nu} = \begin{pmatrix} a & & \\ & b & \\ & & c \\ & & & \ddots \end{pmatrix}$ w/ $a, b, c, \ldots > 0 \Rightarrow$ Euclidean Signature

pseudo-Riemannian $\quad g_{\mu\nu} = \begin{pmatrix} -a & & \\ & b & \\ & & c \\ & & & \ddots \end{pmatrix}$ w/ $a, b, c, \ldots > 0 \Rightarrow$ Lorentzian Signature